INTERACTING HETEROGENEOUS AGENTS PRODUCE ENDOGENOUS INEQUALITY

MATTHIAS GREIFF, STEPHEN KINSELLA, AND EDWARD J. NELL

Abstract. We model an abstract economy of locally interacting heterogeneous agents in four markets, to understand the generation of power law-type distributions of income inequality and firm size in advanced societies. We model a macroeconomy with national accounts built from the interactions of agents (workers, capitalists, bankers, and the government) in time through product, labour, bond, and money markets. We show that, without any restrictions on the type of interaction agents can make, and with asymmetric information on the part of capitalists and workers in this economy, power-law dynamics with respect to firm size and income can emerge from simple multiplicative processes originating in the labour market. Using a new data set, we use only one free parameter to fit the models to firm size and income data for Ireland from 2000 to 2006.

1. Introduction

> [W]hat markets do ... is to generate the pressures that increase productivity ... these pressures bring about innovations, organizational innovations as well as new technologies, which markets then diffuse throughout the system by the force of competition. Market adjustment—the price system—mobilizes the profits to underwrite the investment in these innovations, making the diffusion possible. This means that markets pick winners and losers, which is, indeed, a rough sort of allocation
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> —[11, pp. 6–7].

Economics has studied 'fat tailed' or power-law distributions which characterise firm size and income distributions since the 19th Century [9, 12, 3]. The existence of these power laws[1] has also been demonstrated in the literature in recent work [5, 16, 10, 15]. Models of interaction between heterogeneous agents which generate these dynamics are also beginning to become widespread [2]

In this paper we claim that in a market system characterised by decentralised traders interacting locally with other traders, and globally as members of a 'sector',

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[1]Generally of the form $Pr(k) = \frac{k^{-\gamma}}{\epsilon(\gamma)}$, where k is a positive integer measuring asset returns. $Pr(k)$ is the probability of actually observing k, γ is the power law exponent, and $\epsilon(\gamma)$ is the Riemann zeta function defined as $\sum_{k=1}^{\infty} k^{-\gamma}$.

[2]See [7] for a survey of these models.

rewards accrued by the successful benefit the rewarded, losses hurt the losers, and in so doing, markets generate income inequality and firm size distributions endogenously. We show this using a four sector macroeconomic model comprised of firms, households, banks, and a government. We

We measure income inequality by the construction of a histogram of overall earnings at a point in time, from which we derive the slope of the distribution, inferring greater and lesser income inequality using the standard Gini measurement. Losses are induced by market trading of heterogeneous agents for goods, better jobs, and risky assets, with each agent in our model having initially randomly allocated resources, opportunities, and abilities, which define a reservation wage.

The reservation wage is keyed to an agent's productivity[14], so in labour market bargaining under asymmetric information, each worker tries to bargain for a wage above their reservation wage. The corresponding increase or decrease in firm-level productivity causes true reservation wages to be revealed over time.

The corresponding gains from trade in goods (there is a homogenous capital good) and services (there is both labour and rental income generated in this model) go to those who 'won' at each point in the trading sequence, creating the 'allocative' market alluded to in the quote above.

Winners win, and losers lose, although the loss does not necessarily 'kill' them, thanks to a government transfer to the unemployed. The system carries on as an allocative mechanism—not of resources, but of relative wealth, and the result is a complex system [3].

This model captures several stylised facts pertaining to income distribution and firm distribution [1, 2, 4, 13]:

(1) Economic agents are differently abled, for many reasons. It is natural to model this difference as differences in natural ability, access to resources, and opportunities[4].

    A good example might be the naturally gifted child from who can go to a private school because of their parents' wealth and who is accepted for a scholarship to a prestigious university, who sells their labour for a higher amount than others in the labour market and so 'wins' in our schema.
(2) Wealth, in the form of claims to units of a homogenous capital good, is transferred from agent to agent in a bidding process which we can see.
(3) Those agents that do not find jobs in the labour market do not die: a 'dole' is provided for them by a government, allowing them to survive until the next period. This creates the 'tail' of the distribution we see in empirical studies of income distribution [16, 6] across both developed and undeveloped countries.
(4) Gains made in the labour market and gains made in the capital market affect the individuals inside the model very differently.
(5) We see persistent dispersion of wealth in this model, which we can tune for using what we call a 'carpe diem' coefficient.

[3]where a complex system is defined as the simple macroscopic regularity resulting from the interaction of many agents in decentralised exchange [8]

[4]The main study upon which we base the simulation part of our analysis is D.G. Champernowne's (1972) *The Distribution of Income between Persons.* Champernowne's objective is to produce a model where initially randomly seeded economic agents produce income through a repeated stochastic process, modeled beautifully with Markov chains, which tends towards a (Paretian) power-law distribution in long periods of this process.

2. Description of model and data

The model the interaction of four sets of agents: one person households, firms, a government, and banks. We allow workers and capitalist to meet in a labour market.

To make such a complicated interaction tractable by simulation, each actor is characterised by three randomly allocated scalars: resources, opportunities and ability, and one parameter, which we call the *carpe diem* parameter. Workers are, in general, heterogenous with respect to their skills $0 < \theta < 1$, and their output y, produced by each worker given their work effort n will be given by the product, $y = \theta n$, which we call effective labour. This is represented in the following schema.

2.1. Resources, Opportunities and Abilities.

Each productive unit, formalized as a cellular automaton in our toy economy, is delimited by the following criteria.

Definition 2.1 (Resources)**.** A resource is an initial endowment of productive capacity, labour or unit of a good that is augmented by the different processes of the economy. For households, resources are stocks of goods and availability of labour of a given quality, initially randomly distributed. For firms, resources are stocks of goods and availability of demanders. For banks, resources are labour, supplies of loanable funds and liabilities expressed as credits to the firms and households. To what degree these resources are used by the agents in the system is determined by the 'carpe diem' parameter—see definition 4.

Definition 2.2 (Opportunities)**.** An opportunity is a randomly occurring event that any of the economic agents can either take advantage of or not to advance their holdings. Examples are the presence of available opportunities for a firm to buy cheap or the sudden appearance of a new customer base. An example of opportunities in labour markets are opportunities for new contracts at different wages.

Definition 2.3 (Abilities)**.** An ability is an endogenous quality of each economic agent, which can be modified with investment over time. The household is endowed with a certain quality of labour, dl, which augments the

Definition 2.4 (Carpe Diem)**.** The carpe diem parameter, $0 \leq c \leq 1$, measures the propensity of the firm, household or bank to take advantage of the opportunities, resources and abilities afforded them in each period. Randomly set but binding through time for each actor. This parameter is our tuning fork for the simulations that comprise the better part of the study.

2.2. Data.

Our model is calibrated for Income and Firm Size data for Ireland from 2000-2006. The firm size data is a new data set not previously explored, provided by Forfas, the Irish national competitiveness agency. The income distribution data comes from the Irish Central Statistics office, and also covers the period 2000-2006.

3. Conclusion

In this paper we develop a model of endogenous inequality, based on the idea that simple parameters applied multiplicatively over many generations can give us a good idea of how the distribution of income between persons has evolved up to now. Our model is Borgesian in temperment, and in the vein of Champernowne

and Gibrat, addressing inequality as a multiplicative phenomenon. We apply an agent-based model of the transformation of a society from material equality in initial conditions to distorted income distributions. We show how and where the model was losing it's equality, and showed a redistributive policy to counteract it. Someone has already thought of giving the poor money, however. The effects of 'tuning' this inequality distribution are also shown.

This is a transformational growth theory of markets, writ large. If inequality is generated by the Market (with a capital 'M'), then 'class' is just the name you give to a socially generated partition of certain collections of certain bins in the histogram of overall earnings at some period of time. In this study, we aver that markets do *not* exist primarily to allocate resources efficiently markets represent the ebb and flow of goods and services to winners and losers over time. Through competition in all its forms, markets generate surpluses that go to the successful, who carry these gains forward through time and use them to their own advantage. These knock-on effects alter the distribution of wealth in the system, especially when we take account of the presence of inter-sectoral feedbacks. Trading behaviour makes markets unstable because of the ever-present cost pressures from competing economic entities, which, although richer traders are partially insulated from these cost problems in the short run, will affect all members of the sector eventually.

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Institute for Institutional and Innovation Economics, University of Bremen, Germany
E-mail address: greiff@mac.com

Department of Economics, Kemmy Business School, University of Limerick, Ireland
E-mail address: stephen.kinsella@ul.ie

Department of Economics, New School for Social Research, New York, NY, USA
E-mail address: ejnell@aol.com